Survey Scale Forests

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Latent Variable Modeling

Definition:

Latent Variables are **unobservable phenomena** like

- creativity, (Jauk et al., 2014)
- social anxiety, depression, (Prenoveau et al., 2011)
- psychopathic personality, (Drislane & Patrick, 2017)
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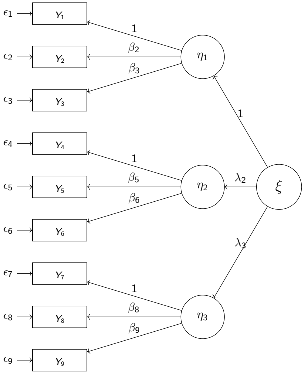
Example:



Causality in Latent Variable Models

Causal relationships expressed in the form of deterministic, *structural relationships* (Bollen, 1989; Pearl, 2009):

$$Y_i = f_i(\xi_i, \zeta_i) \;\; \forall \;\; i = 1, \ldots, m. \qquad (1)$$

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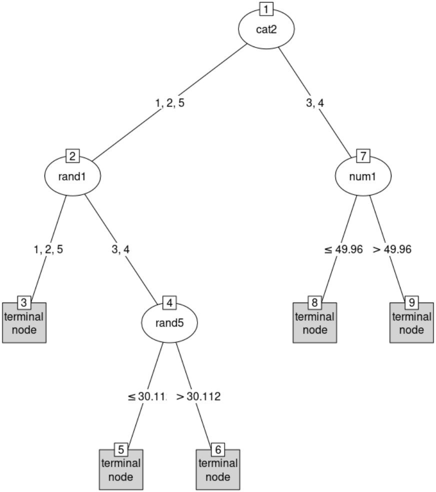
- Direction of Influence

Goal

How can we use latent variable modeling together with **machine learning techniques** to **detect subgroups** in which the assumed model is **not conditionally causal** and exclude them to be able to **estimate valid latent variable scores**?



Conditional Inference Tree in SC Forest

- Reducing parameter heterogeneity by using *fitted model scores* as outcome:

$$\psi(y_j, \theta) = \left(\frac{\partial F_{ML}(y_j, \theta)}{\partial \theta_1}, \ldots, \frac{\partial F_{ML}(y_j, \theta)}{\partial \theta_k} \right), \ \forall\, j = 1, \ldots, n. \quad (2)$$

- Unbiased selection of covariate used for splitting Z_r^*:
  - permutation-based association test between covariates and outcome $\rightarrow$ scale of outcome & covariate irrelevant for the test result!
  - Test $H_0^r : D(\psi | Z_r) = D(\psi)$ for all covariates $r = 1, \ldots, R$, so that global hypothesis test is $\bigcap_{r=1}^{R} H_0^r$
  - If the global hypothesis not rejected $\rightarrow$ algorithm stops splitting

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Predict scores:

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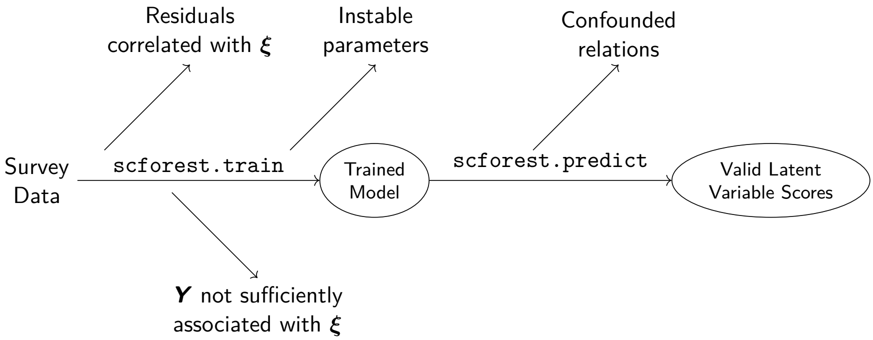
3. Compile results across all iterations

Survey Scale Forest in Practice



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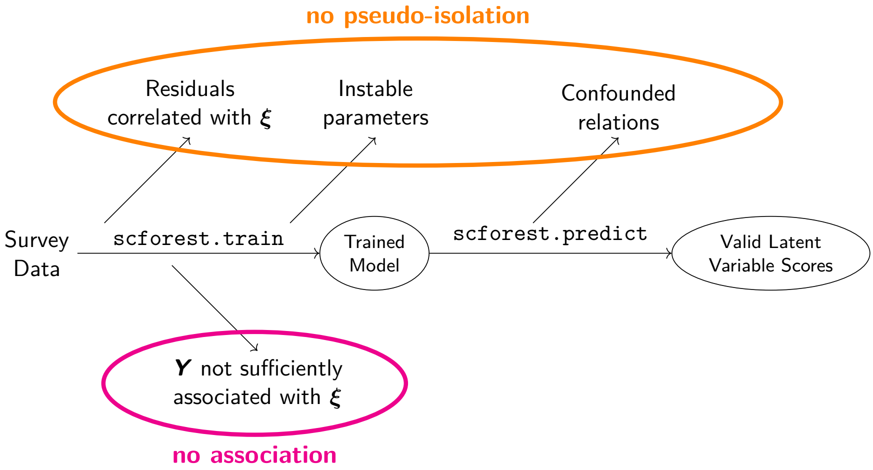

Conclusion

Survey Scale Forest detects two conditions for non-causality,

- lack of pseudo-isolation ($\rightarrow$ confoundedness of relations in model),
- lack of association between $\boldsymbol{Y}$ and $\boldsymbol{\xi}$,

and excludes all subgroups that fulfill these conditions. This way, predicted latent variable scores fulfill criteria for validity although construct may not generally be valid.

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Any questions?

- Contact: classe@ihf.bayern.de
- Want to try the method?:
  R-package `scforest` on GitHub
  github.com/chkern/scforest

Validity

Validity is...

> *"...the degree to which evidence and theory support the interpretations of test scores".* (APA, 2014)

Need for researchers to find **evidence to support proposed interpretation** of item responses.

Four sources of evidence for construct validity (APA, 2014):

Appropriate

- test content,
- internal structure,
- response processes,
- relation to other variables.

$\Big\}\longrightarrow$ Causal Model

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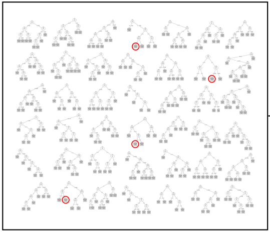
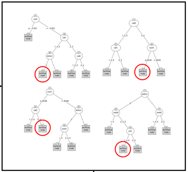
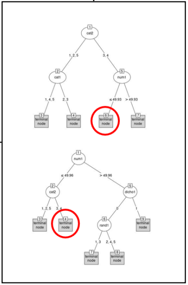
Decision Trees

- Non-parametric machine-learning method
- Recursively partitions covariate space over $\boldsymbol{Z} = \{Z_1, \ldots, Z_R\}$ into set of terminal nodes (*leaves*)
- Reduce outcome heterogeneity
- Usually built on training data and used to predict outcome in test data

Simulation







	id	tree48	tree85	mean
874	1905	-2.2302828	-2.0917307	-2.1610068
3746	1697	-1.8054496	-1.6870492	-1.7462494
3881	1766	-1.7982356	-1.6667754	-1.7325055
1210	1201	-1.6308783	-1.4931119	-1.5619951
2052	1850	-1.6212129	-1.4812004	-1.5512066
602	1485	-1.5977508	-1.4430702	-1.5204105
2407	1613	-1.5548364	-1.4301529	-1.4924947
1648	1694	-1.4915598	-1.3995411	-1.4455505
3406	1764	-1.4859940	-1.3507287	-1.4183614
2846	1244	-1.4722057	-1.3507859	-1.4114958
883	1701	-1.4214872	-1.3055205	-1.3635038
2722	1928	-1.4115237	-1.2983988	-1.3549612
2948	1582	-1.3673506	-1.2803734	-1.3238620
592	1363	-1.3728592	-1.2563205	-1.3145899
567	1976	-1.3369690	-1.2245041	-1.2807365
2057	1535	-1.3092645	-1.1915843	-1.2504244
941	1922	-1.2884267	-1.1832917	-1.2358592
770	1748	-1.2842958	-1.1788328	-1.2315643